4th Workshop on Optimal Control of Dynamical Systems and Applications

Wednesday, 26 February 2025 - Friday, 28 February 2025

Villany

4th Workshop on Optimal Control of Dynamical Systems and Applications

Book of Abstracts

Contents

Plenary talk

Optimal control for a class of linear transport-dominated systems via the shifted proper orthogonal decomposition

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Solving optimal control problems for transport-dominated partial differential equations (PDEs) can become computationally expensive, especially when dealing with high-dimensional systems. To overcome this challenge, we focus on developing and deriving reduced-order models that can replace the full PDE when solving the optimal control problem. Specifically, we explore the use of the shifted proper orthogonal decomposition (sPOD) as a reduced-order model, which is particularly effective for capturing high-fidelity, low-dimensional representations of transport-dominated phenomena. Furthermore, we propose two distinct frameworks for addressing these problems: one where the reduced-order model is constructed first, followed by optimization of the reduced system, and another where the original PDE system is optimized first, with the reduced-order model subsequently applied to the optimality system. We consider a 1D linear advection equation problem and compare the computational performance of the shifted POD method against conventional methods like the standard POD when the reduced-order models are used as surrogates within a backtracking line search.

Wednesday block 1

Fast and memory-efficient optimization for large-scale data-driven predictive control

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Recently, data-enabled predictive control (DeePC) schemes based on Willems' fundamental lemma have attracted considerable attention. At the core are computations using Hankel-like matrices and their connection to the concept of persistency of excitation. We propose an iterative solver for the underlying data-driven optimal control problems resulting from linear discrete-time systems. To this end, we apply factorizations based on the discrete Fourier transform of the Hankel-like matrices, which enable fast and memory-efficient computations. To take advantage of this factorization in an optimal control solver and to reduce the effect of inherent bad conditioning of the Hankel-like matrices, we propose an augmented Lagrangian BFGS-method. We illustrate the performance of our method by means of a numerical study.

Wednesday block 2

Data-Driven Controller Synthesis for Dissipativity: a Dualization-Based Approach

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Over the past years, there has been extensive research in the control systems community on data-based approaches for the analysis and synthesis of dynamical systems. A distinguishing feature of this "new wave"of data-driven methods is the emphasis on solutions and techniques with provable guarantees regarding the (robust) stability and performance of the considered (closed-loop) systems. In this talk, we present results that contribute to this area of research. We present a non-conservative controller synthesis method for discrete-time linear time-invariant systems. The goal of the synthesis is to render the closed-loop system dissipative with respect to a given, generic, unstructured quadratic supply function. Both static state-feedback control and dynamic output-feedback control are considered, with the latter restricted to systems of a specific autoregressive form. The plant model is assumed to be (partially) unknown, but instead we require knowledge of trajectories in the control channel, i.e., the controlled input and measured state/output available for control. It is assumed that these trajectories are corrupted by bounded noise. Replacing a model by the recorded finite length trajectories is what makes the presented approach a data-based approach. The resulting controllers are guaranteed to achieve, in a non-conservative way, the closed-loop stability and the desired dissipativity properties for the entire set of systems consistent with the (noisy) data. The controller synthesis method is based on a convexification procedure that leverages the dualization lemma.

Wednesday block 2

System Identification of LTI Systems With Generalized Orthogonal Basis Functions

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Recently, frequency domain based pole finding methods were developed for LTI systems, which, unlike current popular identification schemes, do not require knowledge of the system order. These methods rely on rational orthogonal expansions of the transfer function and usually require an accurate estimate of the frequency response. This is usually only available in the form of an empirical transfer function, which has a major drawback of being sensitive to noise and prone to numerical errors. In this presentation we propose a novel, end-to-end identification pipeline for LTI systems which incorporates the above mentioned new pole finding schemes, but requires only time domain data to produce the required expansion coefficients. We propose novel ways to compute these coefficients, which allows for noise robust and numerically stable frequency response representations. In addition, the computed Malmquist-Takenaka-Fourier coefficients can be directly used to identify the examined system, resulting in a nonparametric (with respect to system order) and robust identification scheme. To demonstrate the effectiveness of the proposed approach, we present experiments based both on simulated examples and real-life system identification problems.

Wednesday block 2

Rational Prony methods for system identification

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A novel method to identify the transfer functions of single-input-single-output linear time invariant (SISO-LTI) dynamical systems is proposed. The proposed approach uses an operator based generalization of Prony's classical parameter estimation method. In this work, generalized Prony schemes are used to reconstruct the transfer function of the system as a linear combination of rational basis functions. Rational orthogonal expansions of the frequency response are also levaraged to improve the numerical stability of the considered algorithms. It is shown, that the discussed algorithms are robust to noise and unknown system order. In addition to theoretical considerations, the effectiveness of the method is demonstrated through experiments.

Wednesday block 3

LQG Balanced Truncation for Linear Port-Hamiltonian Descriptor Systems

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Linear quadratic Gaussian (LQG) balanced truncation is a model reduction technique for possibly unstable linear time-invariant systems which are to be controlled by an LQG controller. In contrast to the classical balanced truncation approach, the LQG balancing procedure is based on the closed-loop system behavior rather than on the transfer function of the open-loop system. Furthermore, LQG balanced truncation comes with an a priori error bound in the gap metric and yields both a reduced-order model (ROM) for the plant and a corresponding reduced-order controller.

In this talk, we consider the special case where the plant is described by a linear port-Hamiltonian (pH) descriptor system. The pH structure implies many desirable properties such as passivity, which motivates preserving this structure during model reduction. We demonstrate how LQG balanced truncation can be modified such that the ROM is also pH and that this modification still allows for an a priori error bound in the gap metric. Furthermore, we introduce a theoretical approach for exploiting the non-uniqueness of the pH representation to achieve a faster decay of the error bound. This approach is based on a maximal solution of a Kalman-Yakubovich-Popov linear matrix inequality for descriptor systems. Finally, the theoretical findings are illustrated by means of a numerical example.

Wednesday block 3

Row-aware Randomized SVD with Applications

Authors: Davide Palitta[1]; Sascha Portaro[1]

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We introduce a novel procedure for computing an SVD-type approximation of a matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$, $m \geq n$. Specifically, we propose a randomization-based algorithm that improves over the standard Randomized Singular Value Decomposition (RSVD). Most significantly, our approach, the Row-aware RSVD (R-RSVD), explicitly constructs information from the row space of $\mathbf{A}$. This leads to better approximations to Range$(\mathbf{A})$ while maintaining the same computational cost. The efficacy of the R-RSVD is supported by both robust theoretical results and extensive numerical experiments. Furthermore, we present an alternative algorithm inspired by the R-RSVD, capable of achieving comparable accuracy despite utilizing only a subsample of the rows of $\mathbf{A}$, resulting in a significantly reduced computational cost. This method, that we name the Subsample Row-aware RSVD (Rsub-RSVD), is supported by a weaker error bound compared to the ones we derived for the R-RSVD, but still meaningful as it ensures that the error remains under control. Additionally, numerous experiments demonstrate that the Rsub-RSVD trend is akin to the one attained by the R-RSVD even for small subsampling parameters. Finally, we consider the application of our schemes in two very diverse settings which share the need for the computation of singular vectors as an intermediate step: the computation of CUR decompositions by the discrete empirical interpolation method (DEIM) and the construction of reduced-order models in the Loewner framework, a data-driven technique for model reduction of dynamical systems.

Wednesday block 4

Pollution free eigenvalue bounds for the Gramian operator

Author: Luka Grubisic[1]

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In this talk we will present eigenvalue estimates for the solutions of operator Lyapunov equations with a noncompact (but relatively Hilbert Schmidt) control operator. We compute eigenvalue estimates from Galerkin discretizations of Lyapunov equations and discuss the appearance of spurious (non convergent) discrete eigenvalues. This phenomenon is called the spectral pollution. Our main tools, which are of independent interest in their own right, are new improved asymptotic estimates for the eigenvalue decay in the case of control operators of large or infinite rank as well as a rank criterion for determining the part of the spectrum of the discrete Gramian which is converging to the eigenvalues of the full Gramian (pollution free part of the discrete spectrum). We test our theoretical results on a collection of academic prototypes using both finite element as well as spectral element discretizations. We will also give a general overview of the regularity theory for the eigenvectors of solutions of Lyapunov operator equation and its influence on construction high order piecewise polynomial approximations (hp-adaptive finite elements).

Wednesday block 4

The spectrum of periodic isometries on the spaces of continuous functions

Authors: Dijana Ilišević[1]; Chih-Neng Liu; Bui Ngoc Muoi; Ngai-Ching Wong

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The Koopman linearization enables the use of the linear operator theory in the study of nonlinear dynamical systems by switching from a topological dynamical system (K, φ) to a Koopman system $(C(K), T_\varphi)$ consisting of the space $C(K)$ of continuous complex-valued functions on K and the composition operator $T_\varphi \colon f \mapsto f \circ \varphi$ on $C(K)$.

Let $C_0(K)$ be the Banach space of all continuous complex-valued functions on a connected locally compact Hausdorff space K, vanishing at infinity. According to the classical Banach-Stone theorem, surjective linear isometries on $C_0(K)$ are weighted composition operators, that is, of the form $f \mapsto u(\,\cdot\,)\, f \circ \varphi$ for some continuous unimodular function $u\colon K \to \mathbb{C}$ and a homeomorphism $\varphi\colon K \to K$. In this talk, the spectrum of periodic linear isometries on $C_0(K)$ will be described.

Wednesday block 4

G-convergence of m-accretive extensions of Friedrichs operators

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The concept of positive symmetric systems, also known as Friedrichs systems, originated with Kurt Otto Friedrichs in 1958. He demonstrated that this framework encompasses a broad range of initial and boundary value problems for various types of linear partial differential equations. Renewed interest in these systems emerged from advancements in the numerical analysis of differential equations, particularly with the development of abstract Friedrichs systems in Hilbert spaces by Ern, Guermond, and Caplain in 2007. Over the past two decades, the theory has been extensively explored from both theoretical and numerical perspectives, including the homogenization theory for Friedrichs systems introduced by Burazin and Vrdoljak in 2014.

A connection between m-accretive extensions of Friedrichs operator and extensions "with signed boundary map" shall be presented. Then, for such extensions the revisited homogenization theory shall be presented in which we demonstrate that G-compactness can be achieved under significantly weaker conditions than those required in the original study by Burazin and Vrdoljak (2014). This extension broadens the applicability of G-compactness results to equations that exhibit memory effects in the homogenized limit while circumventing the use of compactness techniques employed in earlier approaches.

Plenary talk 2

Gradient-preserving hyper-reduction of nonlinear parametric dynamical systems

Author: Cecilia Pagliantini[1]

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In this talk, we will discuss a hyper-reduction method for nonlinear parametric dynamical systems characterized by gradient fields such as (port-)Hamiltonian systems and gradient flows. The gradient structure is associated with conservation of invariants or with dissipation and hence plays a crucial role in the description of the physical properties of the system. We propose to combine empirical interpolation (EIM) with parameter sampling to approximate the parametric nonlinear gradients mapped into a suitable reduced space. Whenever the evolution of the nonlinear gradients require high-dimensional EIM approximation spaces, an adaptive strategy is performed. This consists in updating the hyper-reduced function via a low-rank correction of the EIM basis. Numerical tests on parametric Hamiltonian systems will be shown to demonstrate the improved performances of the hyper-reduced model compared to the full and reduced models. This is joint work with F. Vismara (TU Eindhoven).

Thursday block 1

Deep Learning Reduced Order Modelling on Parametric and Data driven domains

Authors: Boris Muha[1]; Domagoj Vlah[2]; Iva Manojlović[2]; Martina Bukač[3]

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Partial differential equations (PDEs) are extensively utilized for modeling various physical phenomena. These equations often depend on certain parameters, necessitating either the identification of optimal parameters or solving the equation across multiple parameters to understand how a structure might react under different conditions. Performing an exhaustive search over the parameter space requires solving the PDE multiple times, which is generally impractical. To address this challenge, Reduced Order Models (ROMs) are constructed from a snapshot dataset comprising parameter-solution pairs. ROMs facilitate the rapid solving of PDEs for new parameters. Recently, Deep Learning ROMs (DL-ROMs) have been introduced as a new method to obtain ROM. Additionally, the PDE of interest may depend on the domain, which can be characterized by parameters or measurements and may evolve with the system, requiring parametrization for ROM construction. In this paper, we develop a Deep-ROM capable of extracting and efficiently utilizing domain parametrization. Unlike traditional domain parametrization methods, our approach does not require user-defined control points and can effectively handle domains with varying numbers of components. Moreover, our model can derive meaningful parametrization even when a domain mesh is unavailable, a common scenario in biomedical applications. Our work leverages Deep Neural Networks to effectively reduce the dimensionality of the PDE and the domain characteristic function.

Thursday block 2

Nonlinear Feedback Control of the Fluidic Pinball

Authors: Ali Bouland[1]; Jeff Borggaard[1]

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We develop and present a range of feedback control laws for the fluidic pinball control problem. This control problem seeks to control the vortex shedding behind three cylinders where cylinder rotation is the actuation mechanism. This benchmark problem has been used to demonstrate several machine learning control strategies. In this talk, we present an approach that uses interpolatory model reduction to build a polynomial approximation to the perturbation of the Navier-Stokes flow from the steady-state solution that maps the three control inputs to twenty-four output measurements taken downstream of the cylinders. Using this model, we use polynomial approximations to Hamilton-Jacobi-Bellman equations to create a quadratic feedback control law. Numerical simulations of this feedback law (a closed-loop simulation performed using FEniCS) demonstrate that we can completely stabilize the steady-state solution (i.e. no vortex shedding) over a range of low Reynolds number flows. We will comment on the sensitivity of the controller to boundary conditions.

Thursday block 2

Polytopic Autoencoders and Series Expansions for Nonlinear Feedback Control Design

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Polytopic autoencoders provide low-dimensional parametrizations of states in a polytope. For nonlinear PDEs, this is readily applied to low-dimensional linear parameter-varying (LPV) approximations as they have been exploited for efficient nonlinear controller design via series expansions of the solution to the state-dependent Riccati equation. In this work, we develop a polytopic autoencoder for control applications and show how it improves on standard linear approaches in view of LPV approximations of nonlinear systems and how the particular architecture enables higher order series expansions at little extra computational effort. In a numerical study, we illustrate the procedure and how this approach can reliably outperform the standard linear-quadratic design.

Thursday block 2

Passive feedback control for nonlinear systems

Authors: Tobias Breiten[1]; Attila Karsai[1]

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Dynamical systems can be used to model a broad class of physical processes, and conservation laws give rise to system properties like passivity or port-Hamiltonian structure. An important problem in practical applications is to steer dynamical systems to prescribed target states, and feedback controllers provide a powerful tool to do so. However, controllers designed using classical methods do not necessarily obey energy principles, which makes it difficult to model the controller-plant interaction in a structured manner. To support structured controller design for nonlinear systems, we study modifications of classical control design methods. We combine the concepts of output feedback and optimal control and show that the corresponding controller exhibits passivity properties regardless of the original system structure. Furthermore, under the assumption that the plant is port-Hamiltonian, we state conditions for the controller to be port-Hamiltonian as well. Two dimensional numerical examples based physical models will illustrate benefits and current limitations of the method.

Thursday block 3

Periodic Optimal Control of a Plug Flow Reactor Model with an Isoperimetric Constraint

Author: Peter Benner[1]

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We study a class of nonlinear hyperbolic partial differential equations with boundary control. This class describes chemical reactions of the type "A ⟶ product"carried out in a plug flow reactor (PFR) in the presence of an inert component. An isoperimetric optimal control problem with periodic boundary conditions and input constraints is formulated for the considered mathematical model in order to maximize the mean amount of product over the period. For the single-input system, the optimality of a bang-bang control strategy is proved in the class of bounded measurable inputs. The case of controlled flow rate input is also analyzed by exploiting the method of characteristics. A case study is performed to illustrate the performance of the reaction model under different control strategies. We show that the optimal periodic boundary control improves the yield as compared to the traditional steady operation of a PFR.

Thursday block 3

Optimal Control Strategies for Wine Fermentation and Materials Design: NMPC vs. Bayesian Optimization

Author: Christina Schenk[1]

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Optimizing complex processes is crucial in industries like food production and materials science, where improving product quality and efficiency can drive innovation. Achieving optimal control in these systems requires advanced techniques to handle nonlinearity, uncertainty, and high-dimensionality. In this talk, we will navigate a diverse landscape of approaches encompassing physical model-based, probabilistic, and data-driven solutions, discussing their advantages and drawbacks in the context of real-world applications. We focus on Nonlinear Model Predictive Control (NMPC) with parameter estimation and Bayesian Optimization (BO), comparing their applications in wine fermentation [1] and materials discovery [2]. For wine fermentation, NMPC uses process models to predict and optimize control actions like temperature to lower energy consumption while maintaining product quality. Parameter estimation adapts the model in real-time to account for uncertainties in the fermentation process. In materials design, BO efficiently explores high-dimensional design spaces by building a probabilistic surrogate model, guiding the optimization of material properties with limited experimental data. We will discuss the strengths and limitations of both methods, their integration with lab automation, and their impact on accelerating innovation and improving process efficiency. This talk offers insights into how model-based and data-driven control strategies can be applied to real-world optimization challenges in food production and materials engineering.

Thursday block 4

A simple model of inserting stent to a blood vessel

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A stent is a mesh-like structure deployed in constricted blood vessels. It plays a crucial role in the treatment of various cardiovascular diseases by providing mechanical support to arteries and restoring uninterrupted blood flow. One of key factors for the long-term success of this procedure is the precise deployment of the stent within the vessel, particularly in cases of asymmetric occlusions. In this work, we propose a simplified model based on the independent ring model for the vessel and couple it with the stent model considered as a network of rods using a one-dimensional curved elastic rod model. We establish the existence of solutions for the proposed model and present numerical simulations to demonstrate its applicability.

Thursday block 4

Two-level trust-region method with random subspaces

Author: Andrea Angino[1]

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We present a two-level trust-region (TLTR) method for solving unconstrained nonlinear optimization problems. The TLTR method employs a composite iteration step based on two distinct search directions: a fine-level direction, derived from minimization in the full high-resolution space, and a coarse-level direction, obtained through minimization in a subspace generated via random projection, enabling accelerated convergence. By blending the strengths of full-space and subspace approaches, the TLTR method aligns with multigrid methodologies that leverage hierarchical representations for efficiency. Numerical experiments, with applications in machine learning, demonstrate the efficacy of the proposed method.

Thursday block 4

Numerical analysis of POD and related MOR methods for time-dependent input-output systems

Authors: Mátyás Constans[1]; Zoltán Horváth[1]

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POD and POD combined with interpolation and regression methods like DEIM are popular model order reduction technologies for time-dependent partial differential equations. In this talk, first, we investigate the numerical properties of POD for large-scale advection-dominated problems. We investigate the stability properties of the time-stepping schemes for the Euler equations of compressible fluids. We explore the largest stable stepsizes and conclude with much larger stable CFL numbers. We shall investigate the cone-preservation property for POD. To this aim, we prove mathematical theorems for the cone-preservation of SVD and the best approximating low-rank matrices. Finally, we investigate the consistency property of the DEIM methods and propose a modification that improves the stability of the POD-DEIM procedure for the input-output systems.

Plenary talk 3

Advances in DMD - using the residuals and the Koopman-Schur decomposition

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We present two themes from a recent development of the Dynamic Mode Decomposition (DMD). First, the DMD is presented as a data driven Rayleigh-Ritz extraction of spectral information. This (unlike the mere regression aspect) allows for a better connection with the Koopman operator, and provides better understanding of the dynamics under study. Computable residuals can be used to select physically meaningful eigenvalues and modes, and to guide sparse representation of the snapshot in the KMD (Koopman Mode Decomposition). We believe that this is the proper approach to the DMD as a numerical toolbox for Koopman operator based data driven analysis of nonlinear dynamics. Then, we discuss the problem of ill-conditioned eigenvectors in the non-normal case and show how the recently proposed Koopman-Schur decomposition can be used both in the operator setting and in numerical computation.

Friday block 1

Efficient optimization of parameter dependent structured Lyapunov equation

Author: Ninoslav Truhar[1]

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The talk presents the latest results in the field of viscosity optimization. The first result is the development of an efficient algorithm that leverages new formulas for calculating the trace, as well as the first and second derivatives of the trace, of the associated Lyapunov equation. This approach enhances the precision and computational efficiency of viscosity optimization. The second contribution involves refining the frequency cut-off approximation, which is particularly suited to the types of problems considered in this work. We introduced a novel error bound as an alternative to the standard residual error, offering a new approach that can be efficiently computed with $\mathcal{O}(rn)$ operations for structures with r additional dampers. The effectiveness of these theoretical advancements has been demonstrated through several numerical examples, which validate the practical relevance of our results.

This is a joint work with Ren-Cang Li from University of Texas at Arlington, Department of Mathematics, Arlington, TX, USA.

Friday block 2

Optimal Damping for the Wave Equation

Authors: Petar Mlinarić[1]; Serkan Güğercin[1]; Zoran Tomljanović[None]

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Vibrational structures are susceptible to catastrophic failure or structural damage when external forces induce resonances or repeated unwanted oscillations. One common mitigation strategy to address this challenge is using dampers to suppress the effect of these disturbances. This leads to the question of how to find optimal damper viscosities and positions for a given vibrational structure. While there is extensive research on finite-dimensional second-order systems, optimizing damper placement remains challenging due to the discrete nature of damper positions. In this talk, we investigate the influence of a single damper on an infinite-dimensional system, focusing particularly on the wave equation. We consider two important cases: uniform forcing and boundary forcing. Both cases are analyzed using the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms.

Friday block 2

Stability Analysis and Perturbation Bounds for Gyroscopic Systems

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In this talk, we will consider a linear gyroscopic mechanical systems of the form

$$M\ddot{x}(t) + G\dot{x}(t) + Kx(t) = 0,$$

where the mass matrix $M \in \mathbb{R}^{n\times n}$ and the stiffness matrix $K \in \mathbb{R}^{n\times n}$ are symmetric positive definite matrices, while the gyroscopic matrix $G \in \mathbb{R}^{n\times n}$ is skew-symmetric, i.e., $G^T = -G$, and $x = x(t)$ is a time-dependent displacement vector.

The properties of the system are defined by those of the associated quadratic eigenvalue problem (QEP)

$$\mathcal{G}(\lambda)x = (\lambda^2 M + \lambda G + K)x = 0, \quad 0 \neq x \in \mathbb{C}^n.$$

We will provide an overview of various linearizations and transformations of the QEP that can be used to develop efficient numerical methods for applications such as stability analysis of gyroscopic systems or perturbation theory.

Friday block 2

Absolute and relative perturbation $\tan\Theta$ theorems

Author: Suzana Miodragović[1]

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In this talk, we will discuss the perturbation of a Hermitian matrix pair (H, M), where H is non-singular and M is a positive definite matrix. The corresponding perturbed pair $(\widetilde{H}, \widetilde{M}) = (H + \delta H, M + \delta M)$ is assumed to satisfy the conditions that $\widetilde{H}$ remains non-singular and $\widetilde{M}$ remains positive definite. We derive an upper bound for the tangent of the angles between the eigenspaces of the perturbed and unperturbed pairs. The rotation of the eigenspaces due to perturbation is measured in a matrix-dependent scalar product.

We will demonstrate that the absolute and relative $\tan\Theta$ bounds for the standard eigenvalue problem are special cases of our newly derived bounds. Additionally, we will show how this bound can be applied to stable gyroscopic systems.